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AN ANALYSIS OF A SINGLE LOCATION INVENTORY PROBLEM FOR TWO INTE--ETC(U)

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(6) AN ANALYSIS OF A SINGLE LOCATION INVENTORY PROBLEM FOR TWO INTERCHANGEABLE RECOVERABLE ITEMS

by

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(17) A6

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ABSTRACT

In this paper we examine the interchangeability/substitutability problem for two recoverable items that fail at a single location. We assume the failure processes for each type of item are independent, stationary Poisson processes. We also assume the repair times are exponentially distributed. Furthermore, we assume that the system is a closed system, that is, no items are added to or deleted from the system. We first consider a discrete-time problem and show that this problem is a Markovian decision problem. We then show that for this problem there exist optimal stationary Markov control policies. Next we formulate a continuous time model and show how to find the optimal stationary Markov control policy using linear programming. Unfortunately, this approach is impractical for solving most real problems. Consequently we have established and explored some of the properties that we feel an optimal policy should possess. A discussion of these properties is given in Section IV. Lastly, we will describe a heuristic that can be used to find a good policy. This method is an efficient simulation search method that finds policies having the properties we conjecture an optimal policy should possess.

I. INTRODUCTION

During the past 15 years a substantial amount of research has been conducted related to the management of recoverable items, that is, items subject to repair when they fail [1,2,3,4,5,6,7,8,9,11,12,13,14]. A number of mathematical models have been developed that can be used to determine optimal stockage levels for each recoverable item in both single and multi-echelon systems. Most of the models are based on the assumption that the items are independent. That is, the failure processes among the items are assumed to be independent. Some recent research has been devoted to dependencies in the demand process by recognizing that certain recoverable items have a hierarchical design [1,5,6,13,14]. For these items, the *failure of a recoverable component results in a demand for both a spare* component and the assembly containing the component. However, in all of the models presented to date the replacement rule for a failed component is the same: replace all failed units with a serviceable spare item of the same type.

In this paper we examine a problem that arises when items are sometimes interchangeable or can be substituted for one another during repair. Frequently it is possible to repair a broken assembly using several different types of parts; however, choosing the "correct" part to use to repair the assembly is not based on engineering considerations alone. Using one type of item to complete the repair rather than using a second type of item can cause subsequent parts shortages that can be avoided. This can occur because some items are "more useful" than others. For example, suppose there are only two types of items in the system. The "more useful" item can be used to satisfy a demand for either type of item whereas the "less useful" item can only be used to satisfy demands for its own type of item. Many such interchangeable/substitutable items are found in the Air Force. This is particularly the case for electronic items. In some instances, newly designed items can be used to replace older units when they fail; however, these older units cannot be used to repair a newer generation of an assembly.

In this paper we will examine the interchangeability/substitutability problem for two items that fail at a single location. We assume the failure processes for each type of item are independent, stationary Poisson processes. We also assume the repair times are exponentially distributed. Furthermore, we assume that the system is a closed system, that is, no items are added to or deleted from the system. We first consider a discrete-time problem and show that this problem is a Markovian decision problem. We then show that for this problem there exist optimal stationary Markov control policies. Next we formulate a continuous-time model and show how to find the optimal stationary Markov control policy using linear programming. Unfortunately, this approach is impractical for solving most real problems. Consequently we have established and explored some of the properties that we feel an optimal policy should possess. A discussion of these properties is given in Section IV. Lastly, we will describe a heuristic that can be used to find a good policy. This method is an efficient simulation search method that finds policies having the properties we conjecture an optimal policy should possess.

II. A TWO-ITEM SINGLE LOCATION SYSTEM: A DISCRETE TIME MODEL

In this section we present a discrete-time model for the interchangeability/substitutability problem. We examine this discrete-time model since it yields a particular form for the optimal control policy. In particular, we show that for the associated Markov decision problem there is a stationary Markov control policy which achieves the lowest average back-order level. This result provides the motivation for restricting attention to Markov control policies in the continuous time model developed in the next section.

To simplify the analysis and discussion we restrict our attention to a single location system with only two types of items: type 1 and type 2. The assemblies in which these items are installed are called units; we also assume that there are two types of units and that each unit contains only one item of the types considered. Furthermore, when a type 1 unit or a type 2 unit fails, we assume that it can be repaired with a serviceable type 2 item. Type 1 items can also be used to repair failed type 1 units: however, type 1 items cannot be used to repair type 2 units. For example, two units might be different "generations" of a computer found in a fire control system; the items might be old and new versions of an integrated circuit board found in the computer. The newer version of the circuit board can be used in both generations of the fire control system computer; but, the old generation circuit board is incompatible with the newer fire control system computer.

Let N_i be the number of units of type i and M_i be the number of spare items of type i in the system. Thus there are a total of $N_i + M_i$ items in the system. Let n_{ij} be the number of type i items installed in type j units, and m_i be the number of servicable type i items in spare stock.

Note that according to the substitution rules we have established n_{12} is always zero. Thus the five numbers m_1, m_2, n_{11}, n_{21}, and n_{22} specify the disposition of all items. The number of type i items in repair is given by $N_i + M_i - (m_i + \sum_{j=1}^{i} n_{ij})$, and the number of back-orders associated with type j units is $N_j - \sum_{i=j}^{2} n_{ij}$.

We assume the system operates as follows in the discrete time model. At each time $t = n \cdot (\Delta t)$, $n = 0, 1, \ldots$ certain actions are available. These actions correspond to installing some items currently in spare stock in appropriate units lacking an item. After installation of items, failures may occur. We presume that items fail independently of one another and that each item of type i installed in a unit of type j has probability $\lambda_{ij} \cdot (\Delta t)$ of failing (where (Δt) is small enough so that these numbers do not exceed 1). Next, items which have failed are removed from the units and sent to repair. Items are repaired independently; we presume that each item is repaired during this time interval with probability $r_i \cdot (\Delta t)$.

After this sequence of action-failure-repair we begin again at the new time $(n+1) \cdot \Delta t$ by selecting another action. The system continues to operate in this manner for an indefinite length of time.

The number of backorders during the action-failure-repair cycle is defined to be the number of backorders which exist immediately after the action (unit repair) is taken and before the failures occur.

We wish to choose those actions which minimize the average number of backorders.

The selection of the particular sequencing of events - action, failure repair - was not made arbitrarily. This sequence was selected so that the problem could be formulated as a Markovian decision problem (average cost model). (A discussion of Markovian decision problems can be found in reference 10.)

Notice that for any policy (that is, a specification of the actions to be taken for all possible situations) there is positive probability (actually bounded away from zero) that after one cycle of action-failure-repair all items which were in use will have failed and been repaired. Hence there is a state (namely that with no items installed and all in spare stock) for which every action taken at every state gets to that state in one step with probability greater than or equal to $\beta > 0$. By Ross [10] (Theorem 6.17, Corollary 6.20, and the remarks following Corollary 6.20) there is then an optimal stationary Markov control policy. This policy can be computed by a technique involving linear programming; this technique is adapted to the continuous-time model developed in Section III.

III. A TWO-ITEM SINGLE LOCATION SYSTEM: A CONTINUOUS TIME MODEL

We now consider the continuous time model corresponding to that of Section II. The notation describing the numbers of units and items and the state of the system remain the same.

We now suppose that the failure times of type i items installed in type j units are independent exponentially-distributed random variables with mean $1/\lambda_{ij}$, and that the repair times are independent, exponential random variables with mean $1/r_i$. As before, the measure of performance of the system is the average number of backorders. Motivated by the results of the previous section we shall consider only stationary Markov control policies.

Since it would seem unreasonable to allow backorders for type i units if there are type i items in spare stock we consider the installation of type i items in type i units to be automatic and not subject to control. Thus the actions available involve only the installation of type 2 items in type 1 units. We shall compute the optimal stationary Markov control policy which takes action only when the system changes state due to an item failing in service or being returned from repair. Finally, we allow only the substitution of one type 2 item into type 1 units at a time.

When the process jumps to a new state there are (possibly) two actions available: do nothing, or install a type 2 item in a type 1 unit. Of course, if there are no type 2 items available or no backorders associated with type 1 units, then there is only one action available: do nothing.

Following Ross [10] we allow randomized actions; thus corresponding to state S there are two numbers $P_S^{a_1}$ and $P_S^{a_2}$ (non-negative and summing to 1) giving the probabilities of selecting action a_1 (= do nothing) or a_2 (= put one type 2 item in a type 1 unit). These P's completely specify the control policy.

Once these P's are specified the process which results is a stationary Markov process. In fact, if we consider the process which specifies the state the system most recently jumped to and the action taken there, this process is a Markov chain and we can compute its stationary transition probabilities and then the average cost.

Let Z_S^a be the equilibrium (or stationary) probability that this process most recently jumped to state S and that the action taken there was a. Then we must have $\sum_a \sum_{S_2} \alpha^a_{S_1S_2} Z^a_{S_2} = \sum_a Z^a_{S_1}$ where $\alpha_{S_1S_2}$ $(S_1 \neq S_2)$ is the rate at which transitions to state S_1 will occur if the current state is S_2 and the action taken is "a", and $\alpha^a_{S_2S_2} = -\sum_{\substack{S_1 \\ (S_1 \neq S_2)}} \alpha^a_{S_1S_2}$ so that the column-sums of the α-matrix are zero. (The subscripts may seem reversed in the above. This is because in the usual Markov chain matrix notation the order of vector-matrix multiplication is the reverse of that used in the usual L.P. notation, which we adopt here.)

The cost associated with each state, $C(S,a)$, is the number of back-orders (total for units of type 1 and type 2) associated with state S if action a is chosen.

We wish to minimize the cost-per-unit-time given by $\sum_S \sum_a Z_S^a C(S,a)$ subject to the equilibrium equations $\sum_a \sum_{S_2} \alpha^a_{S_1S_2} Z^a_{S_2} = \sum_a Z^a_{S_1}$ and to the condition $\sum_S \sum_a Z_S^a = 1$

$$Z_S^a \geq 0 .$$

(Note that in some states substitution obviously cannot be performed; for these states we can simply ignore (that is set to zero) the corresponding Z_i^a for a = SUBSTITUTE).

The solution to the above stated L.P. provides the values for the P's (and hence the policy) as follows:

$$P_S^a = P\{\text{take action } a \text{ when in state } S\}$$
$$= \frac{Z_S^a}{\sum_b Z_S^b},$$

whenever $\sum_b Z_S^b \neq 0$. When $\sum_b Z_S^b = 0$, state i is never reached by the controlled system and thus it makes sense to leave the action chosen there undefined.

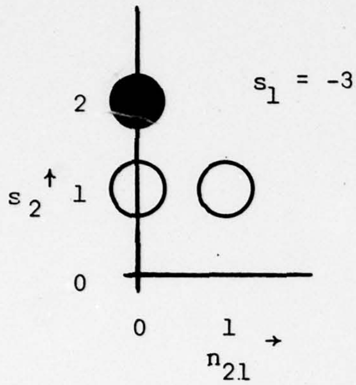
Suppose T represents the number of realizable states in the system (a state is realizable whenever $\sum_a Z_S^a > 0$). Then the above optimization problem has T equality constraints and T basic variables (note there is one redundant constraint). If state S is realizable, then $Z_S^a > 0$ for at least one action a . Consequently at least one Z_S^a is positive for S=1, ..., T . This implies that for each state S , Z_S^a can be positive for only one action a . Thus P_S^a must be either 0 or 1 .

A program was written to generate the input for a linear programming package (MPSX). The output was then submitted to MPSX and (in almost all cases) an optimal strategy was obtained.

To present the results of the computations we notice that under our control policies the number of variables necessary to describe the state of the system can be reduced from the five above to three as follows: Since we never allow backorders for type i units if there is serviceable spare stock on hand; $N_i - \sum_{j=i}^{2} n_{ji}$ and m_i cannot both be positive. We thus set $s_i = m_i - [N_i - \sum_{j=i}^{2} n_{ji}]$, which is the <u>net inventory</u> of items of type i (it is negative if there are backorders associated with type i units).

The variables s_1 , s_2 and n_{21} then describe the state of the system.

Finally, since at each state the control policy merely specifies "substitute" or "don't substitute" it suffices to graph the set of states S at which one would perform a substitution. Graphs of the optimal "substitution set" S for several situations are given in Figure 1, Figure 2, and Figure 3.



$N_1 = 5$

$M_1 = 2$

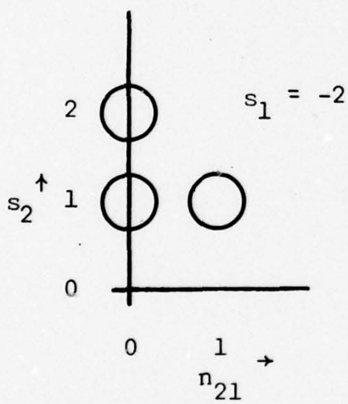
$N_2 = 5$

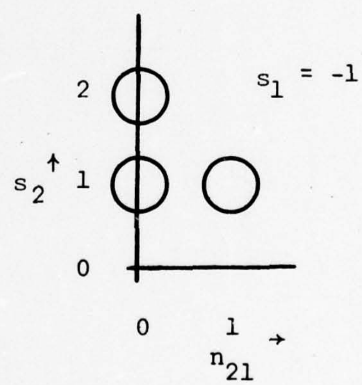
$M_2 = 2$

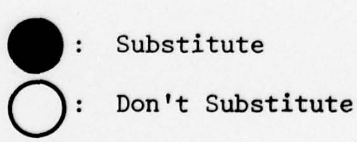
$\lambda_{11} = \lambda_{21} = \lambda_{22} = .17$

$r_1 = r_2 = 1.$

E(Backorders) = .1223

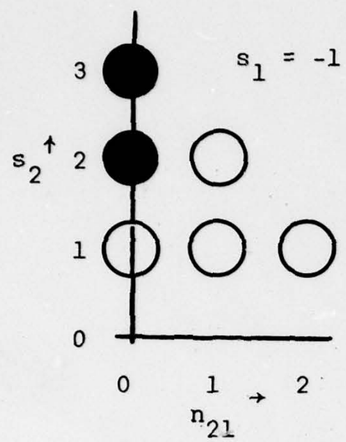






Example A. Optimal Policy

Figure 1.



$N_1 = 4$

$M_1 = 2$

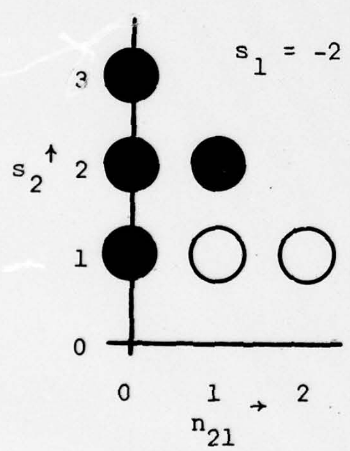
$N_2 = 4$

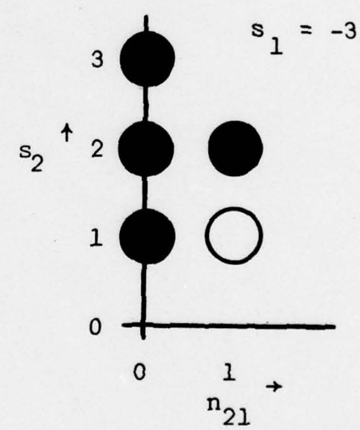
$M_2 = 3$

$\lambda_{11} = \lambda_{21} = \lambda_{22} = .125$

$r_1 = r_2 = .25$

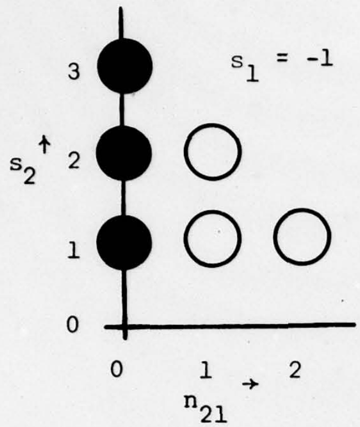
E(Backorders) = .4558





Example B. Optimal Policy

Figure 2.



$N_1 = 4$

$M_1 = 2$

$N_2 = 4$

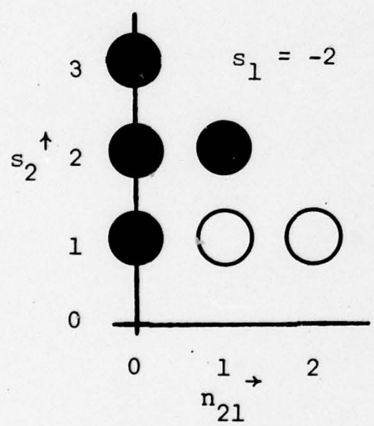
$M_2 = 3$

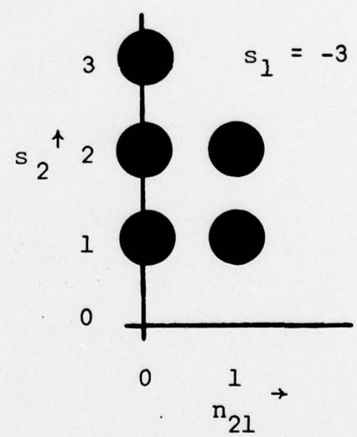
$\lambda_{11} = .125$

$\lambda_{21} = \lambda_{22} = .1$

$r_1 = r_2 = .25$

E(Backorders) = .3321





Example C. Optimal Policy

Figure 3.

IV. THE FORM OF AN OPTIMAL POLICY

To enable us to find approximations to an optimal policy, we examined the structure of the set of states, S , from which a type 2 item should be substituted into a type 1 unit. Clearly

$$S \subseteq \{(s_1, s_2, n_{21}) : s_1 < 0, s_2 > 0\}.$$

Every such subset gives rise to a control policy; however, there are three properties (monotonicity relationships) that one might expect the optimal subset (S_{opt}) to have. Willingness to perform a substitution depends on the number of type 1 units out of service, the number of type 2 items in spare stock, and the number of type 2 items already installed in type 1 units. Intuitively, for fixed s_2 and n_{21} , as s_1 decreases there is at least as great a need to substitute type 2 parts in the type 1 units. We express this as

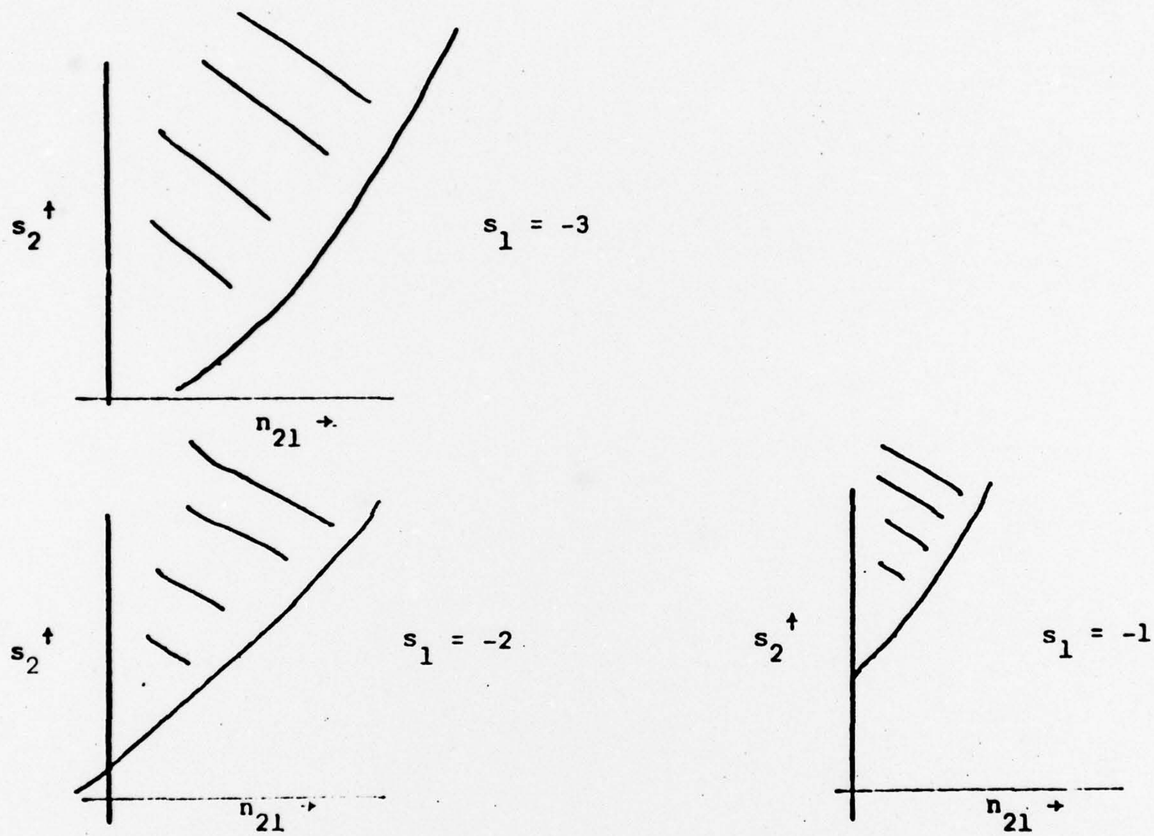
MR1) $(s_1, s_2, n_{21}) \in S_{opt}$, $s < s_1$, implies $(s, s_2, n_{21}) \in S_{opt}$.

Similarly, a greater supply of type 2 spares should imply an equal or greater willingness to substitute. This property is expressed as

MR2) $(s_1, s_2, n_{21}) \in S_{opt}$, $n < n_{21}$, implies $(s_1, s_2, n) \in S_{opt}$.

Since S is a set in three-dimensional space we can draw its graph in sections; thus if we let each section correspond to a fixed value of s_1 we would expect the graph to look like that of Figure 4.

In each optimal strategy computed using the linear programming method described in Section III the properties MR1)-MR3) held. On the basis of this conjectured form of the optimal policy we developed a procedure to examine states for possible inclusion in S .



(The shaded region is the set of substituting states.)

Expected Form of an Optimal Policy

Figure 4.

V. APPROXIMATIONS TO AN OPTIMAL POLICY

To find a good approximation to the optimal substitution policy we employed a search technique which proceeds as follows: begin with S empty (i.e., with a policy which allows no substitution); repeatedly consider adding one point at a time to S by comparing the performance of the system using the augmented S with the current policy; then add the point if the level of performance is higher.

The first states considered are those for which $s_1 = -1$. In this set, the most likely candidate for membership in S is $(-1, M_2, 0)$; i.e., allow substitution when there is a type 1 unit out of service only if there are M_2 type 2 items in stock and none already installed in type 1 units. If this state is added to S, s_2 is successively decreased by one unit as long as the inclusion of $(-1, s_2, 0)$ improves the policy. Then n_{21} is incremented by 1 and $(-1, M_2-1, 1)$ is considered (note, if $n_{21} = 1$, then at most M_2-1 serviceable spare type 2 items can be in stock). Again, s_2 is decreased until there is no further policy improvement. We then increment n_{21} again. After all appropriate states of the form $(-1, s_2, n_{21})$ have been added to S, those for which $s_1 = -2$ are considered in a similar fashion. We take advantage of property MR1) to include automatically in S each state $(-2, s_2, n_{21})$ such that $(-1, s_2, n_{21})$ has already been added to S. This significantly reduces the number of comparisons which have to be made.

An alternate search pattern was considered in which $(-N_1, M_2, 0)$ is the first state examined for inclusion in S. In this state all type 1 units are out of service and all type 2 items are in spare stock. Intuitively, this is the state from which one would be most likely to allow substitution and hence an appropriate starting point for the search. The disadvantage of this approach was that MR1) could not be used to increase the computational efficiency, and hence the first method was used.

The search technique involved being able to compare the performance of two policies. We developed two methods for this: one, analytic, which gave an exact number for the expected backorders under a given policy, and the second, simulation.

In the analytic method we observe that the substitution rules for a given policy and the transition probabilities depend only on the present state, hence the system is a Markov chain. The method consists of generating the steady state equations, finding the equilibrium probability distribution for the chain and calculating the expected number of backorders under the equilibrium distribution. This number can be compared to the expected back-orders for the same policy augmented by one state as required by the search procedure.

A serious disadvantage of this method of policy comparison is related to the number of states in the chain and hence the number of equations to be solved for the equilibrium distribution. The number of states for a system with N_i units of type i and M_i items of type i is

$$(N_i + M_i + 1)(M_2 + 1)(N_2 + M_2/2 + 1) .$$

For example, a relatively small system with $N_i = 10$ and $M_i = 5$ has 1248 states. When the number of units and items grows by a factor of n , the number of states increases roughly by a multiple of n^3 . The time required to solve the equations corresponding to the enlarged system then increases by approximately a multiple of n^9 . We are, therefore, critically limited in the size of the system we can investigate using the Markov analysis to compare policies.

The major advantage of this approach is that the answers are exact and hence two policies can be compared. The similar results obtained by linear programming and by the search technique with analytic policy comparison suggested that the search mechanism is valid and encouraged the development of a more efficient method of policy comparison.

The simulation method, which provides the only tool suitable for the analysis of large systems, performs two simulations, one for each policy and uses the state at which the policies differ as a starting point for the simulation. In one, the substitution of a type 2 part is made, and in the other it is not made. The two simulations are then run until they both reach the same state. This state is not necessarily the one in which the simulations started. If either system reaches the initial state in which the desirability of substitution is being questioned, the substitution is not made.

When the two systems reach the same state the run is terminated and the difference between the number of backorder-days is recorded; call the result (say the number of backorder-days with the extra substitution minus the back-order-days without it) for run i B_i .

We wish to determine $E(B_1)$, since if $E(B_1) > 0$ we should not perform the additional substitution while, if $E(B_1) < 0$ we should. To estimate $E(B_1)$ we computed $\sum_{i=1}^{n} B_i$ and $\sum_{i=1}^{n} B_i^2$; we could then estimate $E(B_1)$ and $V(B_1)$ and construct confidence intervals for $E(B_1)$. Large groups of runs were made; after each group of runs a confidence interval was computed at a selected confidence level. If this confidence interval did not include the origin the procedure was terminated and the appropriate strategy was selected as optimal. Moreover, if the confidence interval did include zero but was shorter than some pre-selected tolerance level, the procedure was terminated and it was concluded that both policies gave nearly the same performance level.

(Actually in our runs this outcome did not occur.) It should be noted that although we were using a sequential procedure we used analysis appropriate for a single sample and that this is not precisely correct. However, in each case the results obtained by simulation were the same as those obtained by analytic comparison. Moreover, the simulation was computationally superior in two respects: the amount of computer time did not grow as rapidly with increased system size as it did for the analytic method, and, perhaps even more relevant in terms of absolute limitations, the simulations did not require the large amounts of storage necessitated by solving such large systems of equations. Substitution sets (S) generated by the search procedure for the same examples presented in Section III are found in Figures 5, 6, and 7.

In conclusion, we make the following observations:

1) the exact solutions obtained through linear programming and the approximate solutions obtained with the search support the assumption that the form of the optimal policy satisfies MR1) - MR3);

2) comparison of exact results and those obtained by a search with analytic policy evaluation indicate that a search is an effective method of policy improvement;

3) the correlation between results of search with analytic comparison and simulation comparison indicates that the simulation method is valid and hence gives us a method for finding an approximation to an optimal policy which can be applied to large systems.

$N_1 = 5$

$M_1 = 2$

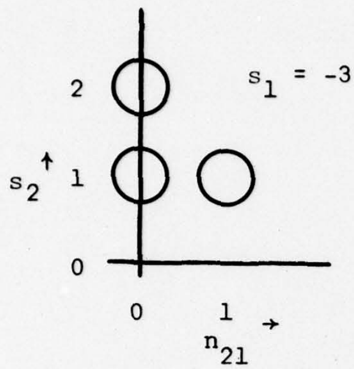
$N_2 = 5$

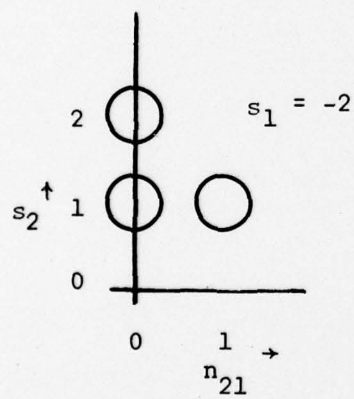
$M_2 = 2$

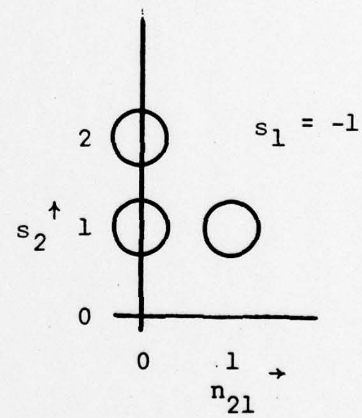
$\lambda_{11} = \lambda_{21} = \lambda_{22} = .17$

$r_1 = r_2 = 1.$

E(Backorders) = .1226

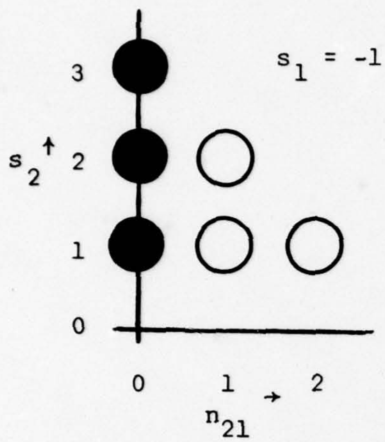






Example A. Policy Obtained from Search

Figure 5.



$N_1 = 4$

$M_1 = 2$

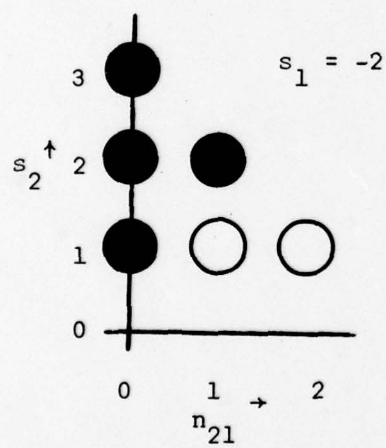
$N_2 = 4$

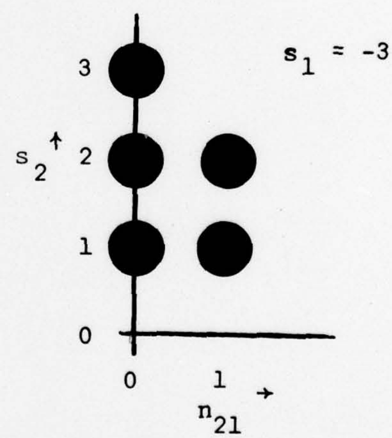
$M_2 = 3$

$\lambda_{11} = \lambda_{21} = \lambda_{22} = .125$

$r_1 = r_2 = .25$

E(Backorders) = .4585





Example B. Policy Obtained from Search

Figure 6.

$N_1 = 4$

$M_1 = 2$

$N_2 = 4$

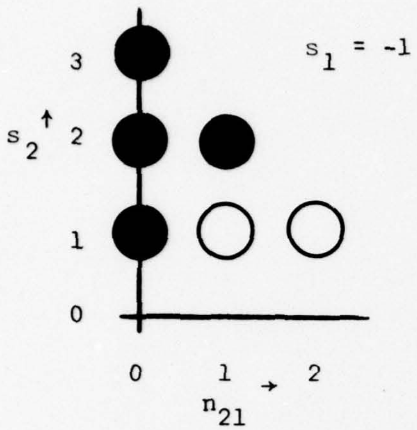
$M_2 = 3$

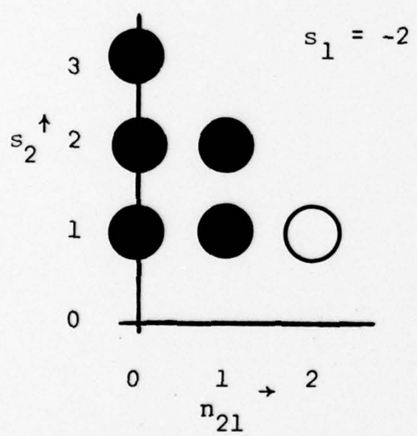
$\lambda_{11} = .125$

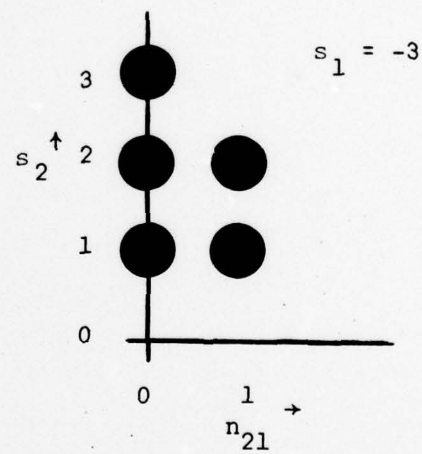
$\lambda_{21} = \lambda_{22} = .1$

$r_1 = r_2 = .25$

E(Backorders) = .3351







Example C. Policy Obtained from Search

Figure 7.

VI. A SUMMARY AND COMMENTS CONCERNING FUTURE EFFORTS

While we now have a method for policy approximation which can be used on large systems, there are several possibilities for further work with this model. Continued investigation of the structure of an optimal policy (i.e., is it linear in any of its variables?) might suggest a reduction in the number of states to be considered for inclusion in S during a search for an approximation to the optimal policy. A more precise comparison of the performance of the exact and approximate solutions should be made to find a balance between reduced backorders and computational accessibility.

In addition, we plan to use the insights obtained through the study of this highly simplified inventory model as a basis for our future efforts to study more complex situations.

Unfortunately, many real-world considerations are not addressed in our simplified model. For example, we have ignored the fact that a) sometimes a family of substitutable items may consist of more than two items, b) items are normally stocked at more than one location, c) the failure and repair distributions may not be stationary, d) the planning horizon may be of such a short duration that infinite horizon models may be inappropriate, and e) the system stock level for each item may not remain constant for an extended period of time. We plan to address many of these issues in our future work.

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In this paper we examine the interchangeability/substitutability problem for two recoverable items that fail at a single location. We assume the failure processes for each type of item are independent, stationary Poisson processes. We also assume the repair times are exponentially distributed. Furthermore, we assume that the system is a closed system, that is, no items are added to or deleted from the system. We first consider a discrete-time problem and show that this problem is a Markovian decision problem. We then show that CONT'

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~~20.~~ for this problem there exist optimal stationary Markov control policies. Next we formulate a continuous time model and show how to find the optimal stationary Markov control policy using linear programming. Unfortunately, this approach is impractical for solving most real problems. Consequently we have established and explored some of the properties that we feel an optimal policy should possess. A discussion of these properties is given in Section IV. Lastly, we will describe a heuristic that can be used to find a good policy. This method is an efficient simulation search method that finds policies having the properties we conjecture an optimal policy should possess.